### Journal
Physical review letters, 66(8)

### ISSN
0031-9007


### Author
Bander, M


### Publication Date
1991-02-01

### DOI
10.1103/physrevlett.66.1001

# Peccei-Quinn Mechanism and Dimension-Six *CP*-Violating Operators

Myron Bander
*Department of Physics, University of California, Irvine, California 92717*
(Received 4 April 1990)

It is shown that the Peccei-Quinn mechanism will, in the large-$N$ limit, remove dimension-six *CP*-violating operators constructed solely out of gauge fields. Such operators have been recently proposed as a source of a possibly large neutron electric dipole moment.

PACS numbers: 11.30.Er, 11.15.Pg, 13.40.Fn

*CP* violations arising from sources other then complex parameters in the Cabibbo-Kobayashi-Maskawa mixing matrix have been recently discussed as an origin for a possibly large neutron electric dipole moment. Weinberg[1] noted that a purely gluonic dimension-six operator, with no small-quark-mass or small-mixing-angle suppressions, would exist in a theory where *CP* violations are due to complex mixing in a multi-Higgs sector.[2] The operator in question is (the normalization of the operator is different from the one used in Ref. 1)

$$\mathcal{O}=f^{\alpha\beta\gamma}G_{\alpha,\mu\rho}G_{\beta,\nu}{}^{\rho}G_{\gamma,\lambda\sigma}\epsilon^{\mu\nu\lambda\sigma}. \tag{1}$$

Here $G$ is the field-strength tensor and the $f$'s are the group structure functions. Origins for such an operator in other models of *CP* violation have been studied by Dai *et al.*[3] and the influence of other dimension-six operators by Boyd *et al.*[4]

The neglect of the dimension-four *CP*-violating operator, $\theta G_{a,\mu\nu}\tilde{G}_a^{\mu\nu}$, is ascribed to a Peccei-Quinn[5,6] mechanism. *It is the purpose of this Letter to point out that this same mechanism will eliminate a large part of the contribution of the dimension-six operator in Eq. (1).* The reason this claim is restricted to only a part of the contribution, rather than to the whole, is that the argument we shall give is valid only to leading order in $1/N$, where $N$ is the number of colors. The Lagrangian incorporating both the dimension-four and dimension-six *CP*-violating operators and the Peccei-Quinn mechanism in the form of an axion field is

$$\mathcal{L}=\cdots+\tfrac{1}{2}\partial_\mu a\,\partial^\mu a-\sum_f\bar{q}_fMq_f-\frac{a}{8\pi f_a}\frac{\alpha_s}{N}G_{a,\mu\nu}\tilde{G}_a{}^{\mu\nu}-\theta\frac{\alpha_s}{8\pi N}G_{a,\mu\nu}\tilde{G}_a^{\mu\nu}-\frac{C}{6N^{3/2}}\mathcal{O}. \tag{2}$$

Here $a$ is an axion field with an axial decay constant $f_a$ while $C$ is the coupling strength of the dimension-six operator of Eq. (1); the summation is over light-quark flavors and the dots represent the quark kinetic energy, the usual electromagnetic interaction as well as the gluon kinetic energy modified by heavy-quark loops. The explicit dependence on the number of colors, $N$, is shown; in the large-$N$ limit all other parameters approach a finite value. The $N^{3/2}$ dependence of the coefficient of $\mathcal{O}$ stems from the fact that $C$ is proportional to $\alpha^{3/2}$.[1] As all *CP*-noninvariant parts of the mass matrix may be either removed by a chiral rotation or shifted to the dimension-four operator, $M$ is real and diagonal.

In the absence of the dimension-six operator a shift of the axion field,

$$a\rightarrow a-\theta f_a, \tag{3}$$

eliminates the $\theta$-dependent *CP* violation; $a=-\theta f_a$ is the value of the axion field at the minimum of the effective potential. In order to determine the vacuum expectation value of the axion field in the presence of the dimension-six operator we need an expression for the effective potential. The effects of the QCD anomaly will manifest themselves if we add

$$\mathcal{L}_a=\frac{-i}{4}\left(\frac{\alpha_s}{8\pi N}\right)\{\mathrm{tr}\ln[\bar{q}_f(1+\gamma_5)q_f]-\mathrm{H.c.}\}G_{a,\mu\nu}\tilde{G}_a^{\mu\nu} \tag{4}$$

to Eq. (2); the above term correctly reproduces the nonconservation of the axial current. Note that $\mathcal{L}_a$ and the term in Eq. (2) that involves the axion field appear together as coefficients of $G\tilde{G}$. These terms will appear together in such a combination in all subsequent effective potentials. It is at this point, when integrating out the gauge fields, that we invoke the large-$N$ limit. Using the color-counting rules[7] the effective Lagrangian becomes[8]

$$\begin{aligned}&\mathcal{L}_{\mathrm{eff}}=\mathcal{L}_0+\mathcal{L}_{\mathrm{em}}-\sum_f\bar{q}_fMq_f+\mathcal{L}_\eta+\mathcal{L}_{\mathrm{CPV}},\\&\mathcal{L}_\eta=-\frac{1}{2}\left(\frac{\alpha_s}{8\pi N}\right)^2\left(\frac{-i}{4}\{\mathrm{tr}\ln[\bar{q}_f(1+\gamma_5)q_f]-\mathrm{H.c.}\}+\frac{a}{f_a}\right)^2t_1,\\&\mathcal{L}_{\mathrm{CPV}}=-\left(\frac{\alpha_s}{8\pi N}\right)\left(\frac{-i}{4}\{\mathrm{tr}\ln[\bar{q}_f(1+\gamma_5)q_f]-\mathrm{H.c.}\}+\frac{a}{f_a}\right)\left(\theta\frac{\alpha_s}{8\pi N}t_1+\frac{C}{6N^{3/2}}t_2\right).\end{aligned} \tag{5}$$

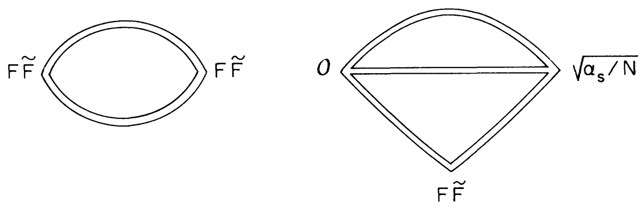


FIG. 1. Color-counting diagrams giving the powers of $N$ for $t_1$ and for $t_2$. Each closed loop gives one power of $N$.

The coefficients $t_1$ and $t_2$ are vacuum expectation values of products of gauge operators:

$$t_1 = \int d^4x \langle G\tilde{G}(x) G\tilde{G}(0)\rangle ,$$
$$t_2 = \int d^4x \langle G\tilde{G}(x) \mathcal{O}(0)\rangle . \tag{6}$$

In Eq. (5) $\mathcal{L}_0$ and $\mathcal{L}_{\rm em}$ are $U(N_f)\times U(N_f)$-invariant and electromagnetic quark Lagrangians, respectively. $\mathcal{L}_\eta$ is due in part to a breaking of the chiral U(1) symmetry by the chiral anomaly and in part to the direct coupling of the axion to the gauge fields; it represents the large mass of the $SU(N_f)$-singlet meson.[9] In Fig. 1 the diagrams giving the powers of $N$ for $t_1$ and for $t_2$ are shown. We see that in the large-$N$ limit $t_1$ behaves as $N^2$, while, as there must be at least one additional three gluon vertex, in $t_2$ this quantity is of order $N^{5/2}$. In this limit all $CP$-violating effects are contained in $\mathcal{L}_{\rm CPV}$. We note once more that for $C=0$ shifting the axion field as in Eq. (3) eliminates the $CP$ violations. For finite $C$, $\mathcal{L}_{\rm eff}$ has a minimum at

$$a = -f_a \left( \theta + \frac{4\pi}{3} \frac{C}{\alpha_s \sqrt{N}} \frac{t_2}{t_1} \right) ; \tag{7}$$

shifting the axion field to this new minimum eliminates the $CP$ violations. This shift is finite in the large-$N$ limit.[10] Equation (7) is the main result of this work. Using similar arguments, higher-dimensional $CP$-violating operators made only out of gauge fields can be eliminated in the large-$N$ limit.

A question arises as to what happens if there is no Peccei-Quinn mechanism and correspondingly no axion field; we assume that $\theta$, the coefficient of the dimension-four operator, is sufficiently small as not to violate phenomenological bounds.[6] In this case a U(1) chiral rotation

$$q_f \to q_f \exp(i\beta\gamma_5) , \tag{8}$$

with $\beta$ given by the coefficient of $f_a$ on the right-hand side of Eq. (7) eliminates $\mathcal{L}_{\rm CPV}$ in favor of a $CP$-violating mass term $-2i\beta\sum_f \bar{q}_f M \gamma_5 q_f$. A subsequent chiral rotation will bring this to the form $-2im\sum_f \bar{q}_f\gamma_5 q_f$, with $m$ proportional to the mass of the lightest quark. Using the rules of "naive dimensional analysis"[1,11] we find that the electric dipole moment of the neutron will be proportional to $m\beta/(4\pi f_\pi)^2$ which in turn is proportional to $mC$. This is to be contrasted with the estimate of Ref. 1 where this contribution was found to be proportional to $C(4\pi f_\pi)$, or about 2 orders of magnitude larger. (A similar situation could have arisen in the case of the dimension-four operator; naive dimensional analysis applied to $\theta G_{a,\mu\nu}\tilde{G}_a^{\mu\nu}$ would yield a contribution to the dipole moment that would be 2 orders of magnitude larger than the one obtained by the correct analysis.[6])

Communications with Professor S. Weinberg are gratefully acknowledged. This work was supported in part by the National Science Foundation under Grant No. PHY-89-06641.

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[10]An interpretation of Eq. (7) is that a shift of the axion field by the negative of the right-hand side of that equation eliminates part of the dimension-four operator; the remaining part cancels, in the large-$N$ limit, the effects of the dimension-six operator.

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